Title: Tessellating with Triangles

Brief Overview:

Any of the theorems that we use in Geometry can be proved strictly with figures; not involving numbers. The most important of these figures is the triangle. This exercise will lead the student to discover several of the common theorems, properties, and postulates using tessellating triangles.

Link to Standards:

- **Problem Solving** Students will demonstrate their ability to solve mathematical problems through the use of the TI-92 Cabri software, or through appropriate drawings.
- **Communication** Students will make conjectures about geometric figures, and state these conjectures using generally accepted language.
- **Reasoning** Students will discover common theorems and postulates of rules of Geometry by experimenting with tessellating triangles.
- **Connections** Students will recognize that theorems and postulates of one figure are applicable to other figures. For example, parallel lines and triangles.
- **Geometry from a Synthetic Perspective** Students will use a model based on triangles to deduce properties.
- **Geometry from an Algebraic Perspective** Students will use rotation and translations to construct congruent figures.

Grade/Level:

Grades 9-12, Geometry

Duration/Length:

This activity will take 2 or 3 days. One day will be used to set up the investigation, and another to make conclusions.

Prerequisite Knowledge:

Students should have working knowledge of the following:

- Definitions of Congruent Angles
- Definition of Straight Angles
- Definitions of Vertical Angles
- Definitions of angles associated with parallel lines
- Definitions of exterior angles and remote interior angles
- Use of the TI-92 (This activity could also be used in demonstration mode; it could even be done on paper.)

Objectives:

Students will be able to:

- become comfortable with the geometry part of the TI-92 (or other software, if used).
- recognize congruences in figures.
- draw conclusions based on graphical representations.
- develop concrete statements based on these conclusions.

Materials/Resources/Printed Materials:

- TI-92 calculator, Cabri Software (if used)
- Pencils/Colored Pencils
- Paper
- Student worksheets
- Teacher Resources Guides

Development/Procedures:

- Introduce each of the required terms - broad definitions are sufficient.

- Develop basic proficiency with TI-92, Cabri software, or other software - if used.

- Complete exercises on following pages.

- Have students write conclusions. In groups, come to consensus on appropriate terminology.

Evaluation:

The students will write a conclusion on what they discovered in this unit.
The students will create a poster showing the tiling and special properties found.

Follow up:

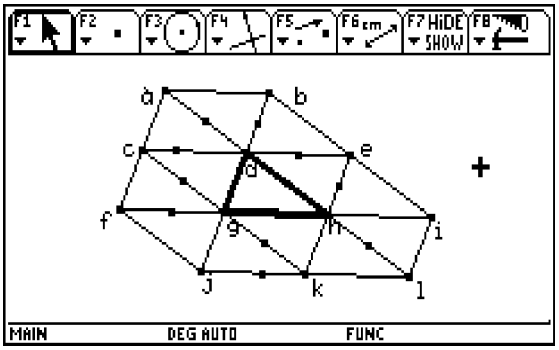
1. After (and during) completion of this exercise, emphasize that the conclusions drawn are independent of the triangle originally drawn. If using the TI-92, 'grab' the vertices of the original triangle to dramatically change the shape, and repeat certain sections. If using paper and pencil, encourage the students to observe other drawings as they work.
2. The congruence of similar figures can be explored.
3. Special rules for right triangles and equilateral triangles can be explored.
4. There are undoubtedly more properties to discover!
5. If the exercise was done on the calculator, encourage students to repeat the exercise on paper, and display.

Authors:

Roger Grondin	David Vogel
Essex High School	Lancaster High School
Essex County, VA	Lancaster County, VA

Teacher Resource Guide

Exploring Triangles
Draw a general triangle.(Δdgh in this drawing.)
Find the midpoint of each side. Using Rotate Around Midpoints, create a drawing similar to one of the drawings below. Use labels to denote vertices.



Notice that there are only three different angles. *You can drag the vertices of the original triangle to verify that the rest of the angles will also change.* Use Measure Angle to verify the congruency of several angles.

In this chart, list the angles congruent to the angles of the original triangle:

∡dhg				∡hgd				∡gdh			

Discuss the definition of congruent figures. Point out corresponding parts. Start with the fact that all of the triangles are congruent to the original triangle, since they are direct copies.

Angles
Consider the intersection of Segments EK and GI. Which angles are formed? (There are four of them.) Which appear to be congruent? Use Measure Angle to verify.
Vertical Angles are the two angles opposite each other. What can you conclude about Vertical Angles? *Vertical angles are congruent.*

Straight Angles
Consider the points f, g, and h. What property do they appear to exhibit? *They are collinear.*
Use Check Properties to verify.

An angle formed by Collinear points is called a Straight Angle. Measure ∠fgh. What can you conclude about the measure of Straight Angles? *The measure is 180.*

The Angles of a Triangle

Δfgh is made up of three angles; ∠fgc, ∠cgd, and ∠dgh
or
∠fgj, ∠jgk, and ∠kgh

Considering the original triangle △DGH, and the three angles above, do you see a relationship between the three angles? *Each of the three angles is congruent to one of the original angles.*

Measure ∡fgh which is called a straight angle. What is the measure of the angle?

So the three angles of the triangle have the same measure as the Straight Angle, which is *180*. So, we can conclude that the three angles of the triangle add up to *180*.

Exterior Angle

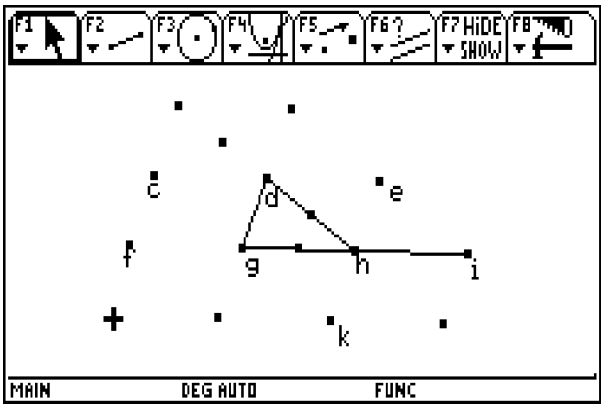
Discuss exterior angles and remote interior angles.

Consider ΔDGH. Extend the segment GI The angle DHI is called an exterior angle of triangle DGH. This angle is made up of which two angles?∡*dhe and* ∡*ehi*

These two angles are congruent to which of the original three angles?

These two angles are called the remote interior angles.

So, the measure of an exterior angle is equal to the *sum of the measures of the remote interior angles?*



Parallel Lines

Consider Segments IL and EK. Do they appear Parallel? Use Check Properties to verify. Which other segments are parallel? List the sets of parallel segments.

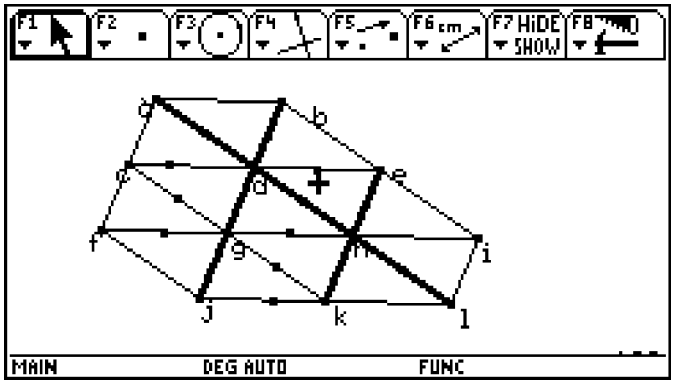
DG	DH	GH

Angles Formed by Parallel Lines
This section will take the longest. A thorough discussion of alternate interior angles, corresponding angles, and same side interior angles is essential.

Consider two parallel lines. Consider a third line that cuts through these two lines. This line is called the transversal.

Notice that eight different angles are formed. How many different measurements are represented?
Three.

On this drawing, mark which angles are congruent to each other.



Teacher Resource 2
Theorems and Postulates:

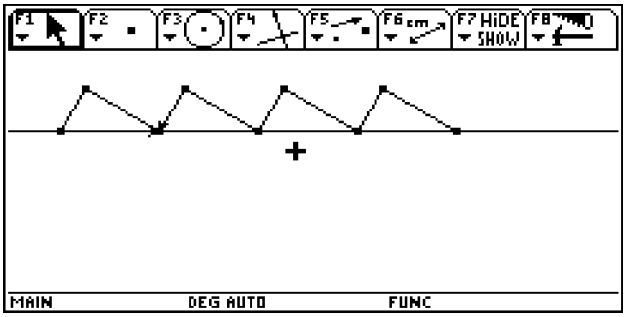
Vertical Angles are Congruent.
The Sum of the angles of a triangle equals 180.
The exterior angle of a triangle is equal to the sum of the two remote interior angles.
If two lines are parallel, then the alternate interior angles are congruent. (Converse)
If two lines are parallel, then corresponding angles are congruent. (Converse)
If two lines are parallel, same side interior angles are supplementary. (Converse)
Corresponding Parts of Congruent Triangles are Congruent.

Extensions:

- Sum of interior angles of quadrilateral, pentagon, n-gon
- Sum of exterior angles of quadrilateral, pentagon, n-gon
- Symmetry concepts
- Similar triangles

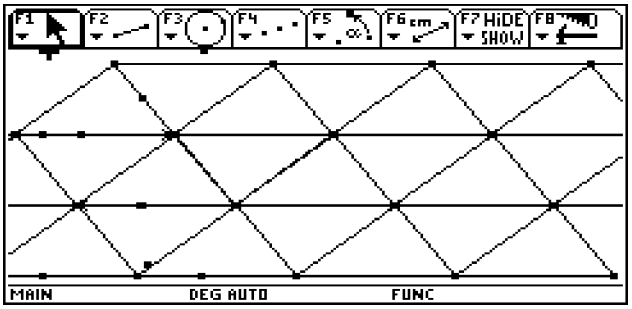
Directions for Use without Calculator

Each student should cut out a triangle from thin cardboard. Color each angle a different color. Draw a line on the paper. Trace the triangle. Color the corresponding angles on the paper. Slide the triangle on the line so that the new triangle is adjacent to the original triangle. Trace and color. Repeat until the edge of the paper. See figure below.



Rotate the triangle so that is relatively 'upside-down'. Notice that it 'fits' within the other triangles. Trace, slide and color. Repeat.

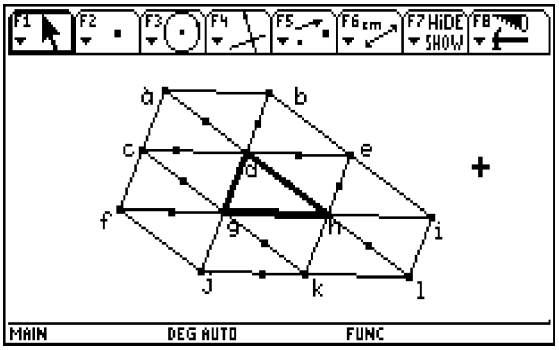
Now move to a second row. Line up the vertices, and complete this pattern below. Continue to color the angles. Tile the entire page.



The students may now complete the rest of the exercise, darkening appropriate segments as they work.

Student Worksheet

Exploring Triangles
Draw a general triangle.(Δdgh in this drawing.) Find the midpoint of each side. Using Rotate Around Midpoints, create a drawing similar to one of the drawings below. Use labels to denote vertices.



Notice that there are only three different angles. Use Measure Angle to verify the congruency of several angles.

In this chart, list the angles congruent to the angles of the original triangle:

∠dhg				∠hgd				∠gdh			

Angles
Consider the intersection of Segments EK and GI. Which angles are formed? (There are four of them.) Which appear to be congruent? Use Measure Angle to verify.

What can you conclude about Vertical Angles?

Straight Angles
Consider the points f, g, and h. What property do they appear to exhibit? Use Check Properties to verify.

Measure ∠fgh.
What can you conclude about the measure of Straight Angles?

Student Worksheet, Page 2: The Angles of a Triangle

Δfgh is made up of three angles; ∠fgc, ∠cgd, and ∠dgh
or
∠fgj, ∠jgk, and ∠kgh

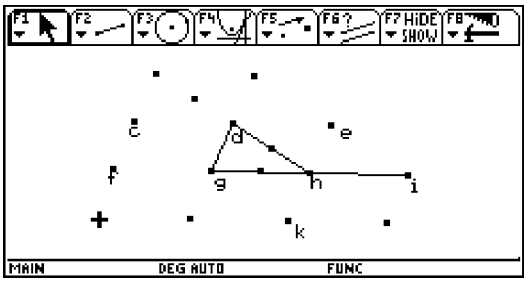
Considering the original triangle △DGH, and the three angles above, do you see a relationship between the three angles?

Measure ∡fgh. What is the measure of the angle?

The three angles of the triangle have the same measure as the Straight Angle, which is _____?

Exterior Angle

Consider ΔDGH. Extend the segment GH. The angle DHI is called an exterior angle of triangle DGH. This angle is made up of which two angles?



These two angles are congruent to which of the original three angles?

These two angles are called the remote interior angles. So, the measure of an exterior angle is equal to the measure of _____?

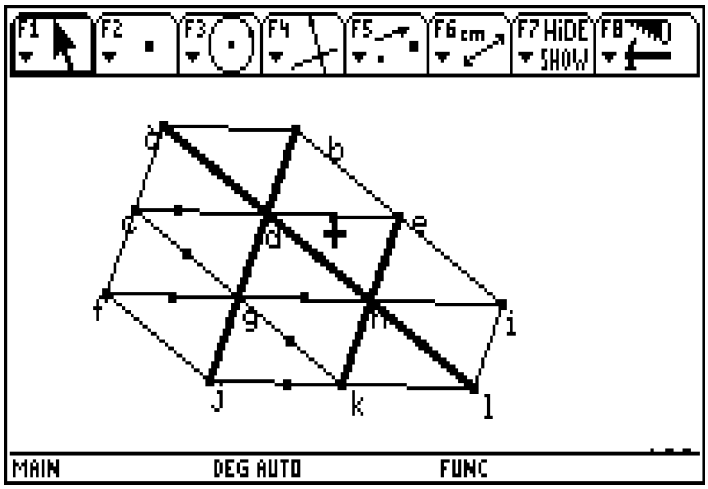
Parallel Lines

Consider Segments IL and EK. Do they appear Parallel? Use Check Properties to verify. Which other segments are parallel? List the sets of parallel segments.

DG	DH	GH

Student Worksheet, Page 3: Angles Formed by Parallel Lines

Consider two parallel lines. Consider a third line that cuts through these two lines. This line is called the transversal.



Notice that eight different angles are formed. How many different measurements are represented?

On this drawing, mark which angles are congruent to each other.

Conclusions

Go through your notes, and write down each conclusion you made in appropriate geometric language.

Create a poster, showing the tesselated triangles, and the representing conclusions made.